## Arithmetic with simple fractions

Z
Work out the answers to these addition and subtraction problems.
Show all your working.
I $\frac{1}{2}+\frac{1}{4} \quad-\quad \frac{3}{4} \ldots \ldots$
$4 \frac{1}{3}+\frac{1}{2}$
$\frac{5}{6}$
$2 \frac{3}{5}+\frac{1}{10}$
$\frac{7}{10}$
$3 \frac{5}{8}+\frac{1}{4}$

$$
0 .
$$

$7 \frac{5}{16}+\frac{3}{8}$
$\frac{11}{16}$
$5 \frac{1}{6}+\frac{1}{3}$
$\frac{1}{2}$
$6 \frac{3}{4}+\frac{1}{4}$
$10 \frac{11}{12}-\frac{1}{4}$
2
$8 \frac{3}{4}-\frac{1}{2}$
$\frac{1}{4}$
$9 \frac{7}{10}-\frac{2}{5}$
$12 \frac{2}{3}-\frac{1}{4}$
$13 \frac{5}{8}-\frac{2}{5}$
$\frac{9}{40}$
II $\frac{11}{12}-\frac{3}{4}$
$\frac{1}{6}$
$\frac{7}{8}$
$16 \frac{14}{15}-\frac{7}{10} \quad \ldots--\frac{7}{30} \ldots \ldots$
$14 \frac{9}{10}-\frac{2}{3}$
$\frac{7}{30}$
I5 $\frac{5}{12}+\frac{3}{10}$
$17 \frac{1}{6}+\frac{5}{9}$
$\frac{13}{18}$
I8 $\frac{11}{16}-\frac{7}{20}$
$\frac{27}{80}$
$19 \frac{2}{11}+\frac{3}{10}$
$\frac{53}{110}$
$20 \frac{5}{7}-\frac{3}{11}$
$\frac{34}{77}$
Work out the answers to these multiplication problems. Show your working in full.
2I $\frac{1}{2} \times \frac{1}{4}$
$\frac{1}{8}$
$22 \frac{1}{3} \times \frac{2}{3}$
$23 \quad \frac{3}{8} \times \frac{1}{4}$
$\frac{3}{32}$
$24 \frac{3}{8} \times \frac{2}{5}$
$\frac{3}{20} \ldots$
$25 \frac{3}{8} \times \frac{2}{3}$
...---- $\frac{1}{4}$
$26 \quad \frac{4}{7} \times \frac{5}{8}$
$\frac{5}{14}$
$27 \frac{3}{10} \times \frac{2}{9}$
$\frac{1}{15}$
$28 \frac{2}{9} \times \frac{6}{7}$
$\frac{4}{2!}$
29
$\frac{2}{9} \times \frac{3}{10}$
$\frac{1}{15}$
$30 \frac{4}{11} \times \frac{11}{7} \quad \frac{4}{7} \ldots$
31
$\frac{2}{3} \times \frac{2}{5}$
$\frac{4}{15}$
32
$\frac{7}{12} \times \frac{2}{21}$
....-- $\frac{1}{42}$
$33 \frac{3}{5} \times \frac{2}{3}$
.-- $\frac{2}{5}$
$34 \frac{5}{18} \times \frac{9}{10}$
$\frac{1}{4}$
$35 \frac{9}{20} \times \frac{5}{6}$
$-\frac{3}{8}$
$36 \frac{3}{4} \times \frac{3}{4}$
$\frac{9}{16}$
$37 \quad \frac{3}{4} \times \frac{6}{13}$
$\frac{9}{26}$
$38 \quad \frac{12}{17} \times \frac{2}{3}$ $\qquad$
$39 \frac{3}{20} \times \frac{12}{13}$
$\frac{9}{65}$
$40 \quad \frac{11}{36} \times \frac{42}{55}$ $\qquad$

Work out the answers to these division problems. Remember to show all your working.
$41 \frac{3}{10} \div \frac{7}{2}$
$\frac{3}{35}$
$42 \frac{4}{5} \div \frac{6}{5}$
$\frac{2}{3}$
$3 \quad 43 \quad 16 \div \frac{1}{2}$
46
49
$\frac{2}{3} \div \frac{4}{3}$
$\frac{1}{4}$
$\frac{5}{7} \div \frac{10}{3}$
$52 \frac{2}{15} \div \frac{8}{3}$
$55 \frac{5}{13} \div \frac{10}{7}$
$\qquad$
$44 \frac{7}{9} \div \frac{14}{3}$
$\frac{1}{6}$
$47 \frac{5}{9} \div \frac{5}{9}$
.
$45 \frac{5}{8} \div \frac{10}{3}$
$\frac{13}{18}$
$\frac{1}{6}$
$\frac{1}{4}$
58 $\frac{5}{7} \div 2$ $\qquad$ $\frac{2}{11}$

Y9 23 WORKING WITH FRACTIONS
Arithmetic with mixed fractions

- 

Work out the answers to these addition and subtraction problems.
Show all your working.
I $1 \frac{1}{2}+1 \frac{1}{4}$
$2 \frac{3}{4}$
$23 \frac{1}{2}+1 \frac{1}{10}$
$4 \frac{3}{5}$
$32 \frac{1}{4}+1 \frac{1}{8}$
$3 \frac{3}{8}$
$43 \frac{5}{8}+1 \frac{1}{2}$
$5 \frac{1}{8}$
$52 \frac{2}{3}+1 \frac{5}{6}$
$4 \frac{1}{2}$
$62 \frac{9}{10}+1 \frac{1}{2}$ $4 \frac{2}{5}$
$74 \frac{7}{16}+5 \frac{3}{4}$
$10^{\frac{3}{16}}$
$82 \frac{2}{3}-\frac{1}{6}$
$2 \frac{1}{2}$
$95 \frac{1}{2}-3 \frac{1}{4}$
$2 \frac{1}{4}$
$104 \frac{3}{4}-3 \frac{1}{3}$
$1 \frac{5}{12}$
II $5 \frac{1}{2}-2 \frac{3}{4}$
$2 \frac{3}{4}$
$126 \frac{1}{10}-5 \frac{1}{2}$
$\frac{3}{5}$
I3 $3 \frac{1}{3}-1 \frac{4}{9}$
$1 \frac{8}{9}$
$145 \frac{2}{5}-2 \frac{3}{4}$
$2 \frac{13}{20}$
$157 \frac{4}{7}+2 \frac{1}{2}$
$10 \frac{1}{14}$
$164 \frac{1}{4}-3 \frac{3}{4}$
$\frac{1}{2}$
$17 \quad 3 \frac{8}{9}+2 \frac{2}{3}$
$6 \frac{5}{9}$
$18 \quad 5 \frac{2}{3}-4 \frac{4}{5}$
$\frac{13}{15}$
19 $5 \frac{3}{11}+4 \frac{9}{10}$
$10 \frac{19}{110}$
$20 \quad 8 \frac{1}{4}-5 \frac{2}{3}$
$2 \frac{7}{12}$

Work out the answers to these multiplication problems. Show your working in full.
$211 \frac{1}{3} \times 2 \frac{1}{2}$
$3 \frac{1}{3}$
$221 \frac{1}{5} \times 3 \frac{1}{3}$ 4 -....... $232 \frac{1}{2} \times 1 \frac{3}{7}$ $3 \frac{4}{7}$
$241 \frac{1}{2} \times 2 \frac{1}{3}$
$3 \frac{1}{2}$
$273 \frac{2}{3} \times 4 \frac{1}{2}$
$16 \frac{1}{2}$
$30 \quad 1 \frac{7}{9} \times 1 \frac{7}{8}$
$3 \frac{1}{3}$
$33 \frac{3}{10} \times 7 \frac{1}{2}$
....... $2 \frac{1}{4}$
$363 \frac{1}{5} \times 1 \frac{3}{4}$
......-. ${ }^{\frac{3}{5}}$
45
45.......
$251 \frac{2}{3} \times 1 \frac{3}{10}$
$2 \frac{1}{6}$
$281 \frac{1}{2} \times 1 \frac{1}{6}$
.-...-1 $\frac{3}{4}$
31 $1 \frac{3}{10} \times 2 \frac{2}{3}$
$3 \frac{7}{15}$
$34 \frac{4}{11} \times 3 \frac{1}{2}$
....... $\frac{3}{11}$
$37 \quad 1 \frac{1}{10} \times 4 \frac{2}{3}$ $\qquad$ $5 \frac{2}{15}$
$40 \quad 5 \frac{3}{8} \times 2$
-....... $10 \frac{3}{4}$

Work out the answers to these division problems. Remember to show all your working.
$41 \quad 2 \frac{2}{5} \div 1 \frac{1}{10}$
$2 \frac{2}{11}$
$421 \frac{1}{3} \div 1 \frac{3}{5}$ $\qquad$ $433 \frac{1}{2} \div 2 \frac{1}{4}$
$1 \frac{5}{9}$
$443 \frac{1}{3} \div 1 \frac{1}{6}$
$2 \frac{6}{7}$
$45 \quad 1 \frac{3}{7} \div 1 \frac{19}{21}$ $\qquad$ $467 \frac{1}{2} \div \frac{3}{4}$ $\qquad$
$473 \frac{1}{4} \div 1 \frac{1}{8}$
$2 \frac{8}{9}$
$485 \frac{1}{2} \div 1 \frac{13}{20}$ $3 \frac{1}{3}$
$49 \quad 1 \frac{5}{6} \div 1 \frac{1}{14}$ $1 \frac{32}{45}$
$50 \quad 2 \frac{2}{3} \div 1 \frac{1}{6}$
$2 \frac{2}{7}$
51 $1 \frac{1}{3} \div \frac{5}{12}$
$3 \frac{1}{5}$
$522 \frac{2}{3} \div 1 \frac{1}{9}$
$2 \frac{2}{5}$
$531 \frac{2}{5} \div 1 \frac{1}{13}$
$1 \frac{3}{10}$
$561 \frac{1}{4} \div 1 \frac{3}{4}$
$\frac{5}{7}$
$593 \frac{1}{2} \div 3$
$1 \frac{1}{6}$
$54 \quad 2 \frac{1}{2} \div 1 \frac{3}{7}$
$1 \frac{3}{4}$
$55 \quad 1 \frac{3}{8} \div 1 \frac{1}{10}$
$1 \frac{1}{4}$

$572 \frac{5}{8} \div 2$
$1 \frac{5}{16}$
$586 \div 2 \frac{1}{2}$
$2 \frac{2}{5}$

## Professor Puzzle's fractions

Professor Puzzle has been working with fractions.
The Professor has forgotten to copy the answers from his notepad into the grid. See if you can work out which answer goes with which question.



Y9) 24 RATIO AND PROPORTION

## Working with ratios

Write these ratios in their simplest form, using whole numbers.
I $16: 24$ 2: 3
2 40:30
4:3
3 8:12
$2: 3$
4 25:15
$5: 3$
5 18:24
3: 4
$6 \quad 42: 28$
3:2
7 22:33
2:3
8 36:72
1:2
9 50:90 $\qquad$

Write these ratios in the form I:n.

| 10 | 4:6 | ...1:1.5 | II | $4: 1$ | 1:0.25 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | $25: 60$ | 1:2.4 | 13 | 18:27 | 1:1.5 |
| 14 | $18: 63$ | 1:3.5 | 15 | 15:9 | 1:0.6 |
| 16 | 8:4 | 1:0.5 | 17 | $3: 4.5$ | 1:1.5 |
| 18 | $25: 4$ | ...1:0.16 |  |  |  |

Write these ratios in the form $n: I$.

| 19 | $3: 2$ | $1 \cdot 5: 1$ | $\mathbf{2 0} 3: 10$ | $0 \cdot 3: 1$ |
| :--- | :--- | :---: | :--- | :--- |
| $\mathbf{2 1} 9: 2$ | $4 \cdot 5: 1$ | $\mathbf{2 2}$ | $50: 20$ | $2 \cdot 5: 1$ |
| $\mathbf{2 3} 16: 25$ | $0.64: 1$ | $\mathbf{2 4}$ | $21: 6$ | $3 \cdot 5: 1$ |
| $\mathbf{2 5} 5: 8$ | $0.625: 1$ | $\mathbf{2 6}$ | $27: 20$ | $\ldots$ |

24 RATIO AND PROPORTION

## Working with ratios (continued)

## Solve these problems about ratio.

28 Anita and Ben have counted their moneyboxes, and find that their savings are in the ratio of $4: 5$. They have a total of $£ 11.25$ between them. Calculate the amount of money that each person has. $\qquad$

29 An examination paper is in two sections, with the marks available on Section A and Section B being in the ratio $2: 3$. The paper is worth 60 marks altogether. How many marks are available in each section?

24,36

30 The number of boys and girls at a local school are in the ratio of $5: 4$ respectively. Altogether there are 873 children at the school. Calculate the number of children of each gender.

31 In my vegetable patch I plant cucumbers, tomatoes and peppers in the ratio $3: 4: 5$. There are 27 cucumber plants. Find the number of plants of each type, and hence find the total number of plants in my vegetable patch.

32 The values of my computer, printer and scanner are in the ratio $8: 3: 2$. Altogether the system is worth $£ 1170$. How much is the scanner worth?

33 During my various fishing expeditions last year, the number of pike, perch and rudd I caught were in the ratio $1: 7: 6$. I caught 70 fish altogether. Calculate the number of fish of each type that I caught during the year.

34 Three friends notice that their ages are in the ratio $8: 9: 11$. Their ages add up to 84 years. Find the age of each of the friends.

24, 27, 33

35 The books in a small library are classed as fiction, non-fiction or reference, in the ratios $5: 3: 1$ respectively. The library has 3750 non-fiction books.
a) Calculate the number of fiction books in the library.
b) Calculate the total number of books in the library.

24 RATIO AND PROPORTION

## Direct proportion

Solve these problems about ratio.
You may use either a method based on ratios or an algebraic method.
I The cost of a car journey is proportional to the distance travelled. A journey of 150 miles costs $£ 42$. Find the cost of travelling 400 miles.

2 The mass of a block of metal is proportional to its volume. A block of volume $30 \mathrm{~cm}^{3}$ has a mass of 195 g . Find the mass of a block whose volume is $70 \mathrm{~cm}^{3}$. $\qquad$
3 The profit made by a small mail order company is proportional to the number of orders it receives. Last year it received 120 orders and made a profit of $£ 11000$. This year it received 150 orders. Calculate this year's profit. $\qquad$
4 The time it takes me to repaint a bedroom is proportional to the area of the walls. Last week I repainted my baby daughter's room, which has a wall area of $45 \mathrm{~m}^{2}$. It took me 90 minutes. Next week I am going to repaint my teenage son's room, which has a wall area of $68 \mathrm{~m}^{2}$. Calculate the time it should take me.

5 A recipe for Moussaka includes 2 eggs, 3 aubergines and 450 grams of mince amongst the ingredients. This is a recipe for four people. I want to cook this dish for 10 people. Calculate the amount of each of these three ingredients that I shall require.

6 The number of digital photographs I can fit on a memory card is proportional to the size of the memory. A 4-megabyte card can hold 20 photographs. How many photographs can I store if I upgrade to a 32-megabyte card?

7 The cost of hiring a jet ski is proportional to the time for which I hire it. A 30 -minute hire costs $£ 20$. How much does it cost to hire for:
a) 45 minutes £30
b) 75 minutes?

24 RATIO AND PROPORTION

## Direct proportion (continued)

8 The number of passengers that can be carried on a miniature railway is proportional to the number of carriages. When the train has six carriages it can carry 96 passengers. How many passengers can be carried when there are ten carriages?

160

9 The number of books I can fit on a shelf is proportional to the length of the shelf. I can fit 20 books on a shelf 50 cm long. How many books can I fit on a shelf of length 1.2 metres?

10 The number of bricks in a garden wall is proportional to its length. A section of wall 1.4 metres long contains 42 bricks. Calculate the number of bricks in the whole wall, which is 3.5 metres long.

II A machinist can sew labels on 50 garments in 8 minutes. How long will it take the same machinist to sew labels on 325 garments?

52 mins
12 The time it takes to complete a country walk is proportional to the total distance. A walk of 10 km takes 2.5 hours. How long does it take to complete a walk of 16 km ?

4 hours
I3 The cost of winter sports insurance is proportional to the number of days for which it is required. Insurance for 7 days costs $£ 24.50$. Find the cost of insurance for 10 days.
$£ 35$
14 The cost of holding an engagement party is proportional to the number of guests who attend. A party for 50 guests will cost $£ 1000$.
a) Find the cost of holding a party for 80 guests.
b) In fact I only have a total budget of $£ 1200$. Find the total number of $£ 1600$ guests I could invite, if I have to stay within this figure.

I5 The number of words in a magazine is proportional to the number of pages. An issue with 36 pages contains 32000 words. Work out the number of words in a bumper edition containing 60 pages. (Round your answer to a sensible level of accuracy!)

## Yes or no?

The tables below show how values of $x$ and $y$ are related.
Decide whether the table indicates that $y$ is directly proportional to $x$.

| I | $x$ | 0 | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| YES | $y$ | 0 | 10 | 20 | 30 |
|  |  |  |  |  |  |


| 2 | $x$ | 1 | 3 | 5 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 2 | 2 | 4 | 6 | 8 |


| 3 | $x$ | 1 | 3 | 5 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| NO | $y$ | 7 | 5 | 3 | 1 |


| 4 | $x$ | 1 | 5 | 6 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | YES | $y$ | 2 | 10 | 12 |
|  | 20 |  |  |  |  |
|  |  |  |  |  |  |


| $\mathbf{5}$ | $x$ | 3 | 4 | 8 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | YES | $y$ | 6 | 8 | 16 |
|  | 20 |  |  |  |  |
|  |  |  |  |  |  |


| $\mathbf{6}$ | $x$ | 10 | 20 | 30 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | YES | $y$ | 2 | 4 | 6 |
|  | 8 |  |  |  |  |
|  |  |  |  |  |  |


| 7 | $x$ | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| NO | $y$ | 10 | 11 | 12 | 13 |
|  |  |  |  |  |  |


| 8 | $x$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| NO | $y$ | 1 | 4 | 9 | 16 |
|  |  |  |  |  |  |

9 YES

| $x$ | 0 | 1 | 5 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | 0 | 3 | 15 | 30 |

10
NO

| $x$ | 0 | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | 10 | 8 | 6 | 4 |

The graphs below show how values of $x$ and $y$ are related.
Decide whether the graph indicates that $y$ is directly proportional to $x$.


13

(2)

14


Y9) 25 ROUNDING AND APPROXIMATION

## Decimal places and significant figures

| I | Round these numbers correct to two significant figures. |  |  |
| ---: | :--- | ---: | :--- |
| I | 75489 | $\ldots$ | 2 |

Round these numbers correct to three significant figures.

| 21 | 85281 | 85300 | 22 | 4896 | 4900 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 23 | 35000 | 35000 | 24 | 8062 | 8060 |
| 25 | 75.472 | 75.5 | 26 | 0.25817 | $0 \cdot 258$ |
| 27 | 4.82945 | 4.83 | 28 | 17.034 | 17.0 |
| 29 | 5.0238 | 5.02 | 30 | 639583 | --640000 |
| 31 | 0.00037252 | . 0.0000373 | 32 | 3.1415926 | .-. $3 \cdot 14$ |

Y9 25 ROUNDING AND APPROXIMATION

## Estimation



Estimate the answers to these calculations by rounding all the numbers to one significant figure. Give the final estimate also to one significant figure.
I $23 \times 11$
200
$2362 \div 19$
20
$3269 \times 36$
.... 10000
$48.3 \div 20$
$0 \cdot 4$.-.--
$52374+56$
2000
$672 \times 64$
4000
$7480 \times 57$
30000
$8 \quad 28 \times 32$
900
$916.3 \div 2.1$
10
-........
$10 \quad 14.4 \times 0.13$ $\qquad$

Round the numbers to one significant figure, and hence make estimates for the answers to these problems.

II The average number of strokes taken for a round of golf on a particular course is 72 . On a competition day, 54 golfers take part. Estimate the total number of strokes played.

I2 A mathematics class of 26 pupils has done an exercise of 18 questions. Approximately how many questions does the teacher have to mark?

I3 Each of the questions (from question 12) take about 20 seconds to mark.
Roughly how long will the marking take the teacher?
I4 At a party, I have decided to serve little sausages on sticks. There will be 34 people at the party, and I estimate that they will eat 3 sausages each. The sausages come in packets of 20 . How many packets do I need to buy?

5

I5 Roughly how long should I allow for a journey of 284 miles if I expect to average 48 miles per hour?

I6 There are 11 flowerbeds in my garden, and I reckon each one holds about 23 flowers. About how many flowers is this in total?

## Amaze yourself

In the grid below, some numbers are given to various numbers of significant figures.
Shade in every square that seems to have a two significant figure number in it.
When you have finished, you should have a maze to find the way through.
(It is not a very difficult one!)

| 75 | 3.6 | 0.72 | 0.045 | 0.05 | 72 | 91 | 5.6 | 0.48 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | 6 | 701 | 0.3 | 10.6 | 234 | 6.9 | 0.203 | 48.5 | 15 |
| 3.4 | 0.003 | 3.7 | 11 | 0.61 | 7 | 0.07 | 12.48 | 8.3 | 46 |
| 38 | 79 | 230 | 0.004 | 5.2 | 0.29 | 93 | 45.20 | 0.036 | 2.3 |
| 35 | 789 | 47 | 65.3 | 8 | 1.73 | 0.009 | 83.02 | 506 | 3.7 |
| 0.61 | 5 | 3.5 | 28 | 18 | 82.0 | 3.1 | 9.2 | 4.607 | 99 |
| 63 | 80.4 | 34.82 | 22 | 58.91 | 0.701 | 6.0 | 6 | 601 | 4.3 |
| 17 | 5 | 7.1 | 82 | 6.753 | 0.13 | 15 | 2.7 | 0.274 | 0.36 |
| 48 | 23.46 | 8 | 6285 | 62.85 | 831 | 10.1 | 28 | 67.92 | 29 |
| 37 | 2.1 | 9 | 0.59 | 5.3 | 39 | 9.4 | 2.8 | 17 | 33 |

Now make a similar puzzle for a friend to try.

## Even more misprints

Spokes Card Game Company have produced yet another card game.
Players have to collect a card containing an equation, and a matching card carrying a solution correct to three decimals places.
Here is one example of a card, and a matching solution. (You can check this solution using a calculator.)

$$
x^{2}-7 x+3=0
$$

$$
x=6.541
$$

Unfortunately, no-one checked the cards for accuracy before they were printed, and at least one error has been made. The full set of cards, and solutions, are shown below. Seven of the eight equations will match up correctly with seven of the eight solutions. Which equations go with which solutions? One pair has already been matched up for you.


Use trial and improvement to find a solution to the leftover equation. Can you now see how the misprint occurred?
A solution to $x^{2}+15 x-2=0$ is $0 \cdot 132$, so the 0 was misprinted as 8 .

Y9) 26 LINEAR EQUATIONS

## Solving linear equations

Solve these linear equations. They require one operation only.
| $x+3=10 \quad \ldots-\ldots$
$2 x-6=9$
15
$34 x=24$
6
$4 \frac{x}{3}=2$
6
$5 x-5=7$
12
$6 x+5=-3$
$7 x-2=-8$
-.-.-6
$87 x=63$
.. 9
.-...
$9 \frac{x}{8}=-1$
$108 x=4$ $\qquad$ || $x+8=2$
-6....
$12 x-9=-1$
13 $6 x=42$
7
$14 \frac{x}{2}=6$
12
15 $x-15=16$
31

These linear equations involve two operations. Solve them.
$163 x+1=13$
4.....
$176 x-4=32$
6
I8 $2 x+5=9$
I9 $16-2 x=10 \quad \ldots 3 \ldots$
$205 x+3=48$
9
2| $7 x+16=2$
$223 x-3=0$
.
$23 \frac{x}{2}+5=9$
8-....
$24 \frac{x}{4}-5=3$ 32
$2511 x-4=29 \quad 3$ .....
$2610-2 x=16$
-3....
$277 x-7=-7$
$28 \frac{x}{6}-14=-4 \quad-\quad 60$
$29 \frac{x}{9}-11=-29$
.- 162
$303 x+7=1$
$-2$

These linear equations have $x$ terms on both sides to begin with. Solve them.
3| $6 x+12=3 x+18 \quad \ldots \quad 2 \ldots$
$327 x+14=5 x+22$
4....
$3312 x+1=3 x+10$
....-......
$342 x+3=x+3 \quad \ldots \quad 0$
$35 \quad 11 x-4=2 x+23 \quad \ldots \quad 3$
$369 x-20=5 x+24 \quad \ldots \quad 11 \ldots$
$37 \quad 8 x-24=6 x+2 \quad-13$
$38 \quad 6 x+12=3 x+18 \quad \ldots-\ldots$
$398 x-1=4 x+15$
...-. 4
$40 \quad 2 x+13=5 x+7$ 2

Y9 26 LINEAR EQUATIONS

## Constructing and solving equations

I Three programmes of length $x, 4 x+20$ and $5 x+30$ minutes exactly fit on a three-hour video tape.
a) Explain carefully why this information can be written as the equation

$$
x+4 x+20+5 x+30=180
$$

b) Simplify the equation and solve it. Hence find the length of each programme.

2 A mathematics textbook contains exercises, activities and investigations. There are $3 x$ pages of exercises, $2 x$ pages of activities and $x+24$ pages of investigations. The book contains 360 pages altogether.
a) Write this information as an equation in $x$.
b) Solve the equation to find $x$. How many pages of investigations are there in the book?
$3 £ 100$ is shared out so that Jack gets $£ x$, Tim gets $£ x+10$ and Martin gets $£ x+15$.
a) Write this information as an equation in $x$.
b) Solve the equation to find $x$. Hence find the amount of money each person gets.

4 The angles in a triangle add up to $180^{\circ}$. The angles are $x^{\circ}, 2 x+15^{\circ}$ and $3 x-45^{\circ}$.
a) Write this information as an equation in $x$.
b) Solve the equation to find $x$. Hence find the size of each angle.

5 The angles in a quadrilateral add up to $360^{\circ}$. The angles are $x^{\circ}, 2 x+18^{\circ}$, $3 x+22^{\circ}$ and $4 x^{\circ}$.
a) Write this information as an equation in $x$.
b) Solve the equation to find $x$. Hence find the size of each angle.
$6 x-30=180$
$x=35$, so
$35^{\circ}, 85^{\circ}, 60^{\circ}$
$10 x+40=360$
$x=32$, so
$32^{\circ}, 82^{\circ}, 118^{\circ}, 128^{\circ}$

26 LINEAR EQUATIONS

## Constructing and solving equations (continued)

For each of the following problems form an equation involving $x$, and then solve it. Show your workings on a separate sheet.

6 Annabel thinks of a number, doubles it, and then adds 5 . She ends up with 21. What number did she first think of?

8

7 Shazad thinks of a number, multiplies it by 6 , and then adds 8 . The answer is ten times the number he started with. What number did he start with?

2

8 John and Sally share a lottery win of $£ 500$. Sally receives $£ 150$ more than John's share. How much was Sally's share?
$£ 325$

9 A piece of ribbon 32 cm long is cut into two unequal parts. The longer part is three times the length of the shorter part. Find the length of each part.

10 Mr Smith is four years older than his wife and twenty-eight years older than his son. Their three ages combined add up to a total of seventy-six. How old is Mr Smith?

36

II Jane has three times as many sweets as Beth. If she gave away six sweets, she would have eighteen left. How many sweets does Beth have?

8

12 A rectangle is $x \mathrm{~cm}$ long and 7 cm wide. Its perimeter is 36 cm . Find the value of $x$.

I3 In triangle ABC , the angle A is $x^{\circ}$. Angle B is $10^{\circ}$ bigger than angle A . Angle C is three times as big as angle A. Find the size of angle A.
$\qquad$

14 Carol has $£ x$ in her purse. John has $£ 20$ more than Carol, and Kate has twice as much as Carol. Altogether they have $£ 80$. Find $x$.

15

I5 In a village there are 650 residents, $x$ of whom are women. There are 36 more men than women and 47 more children than women. Find the number of women.

Missing message

| S | O | L | V | I | N | G | E |  | Q | U | A | T |  |  | 0 | N | S |  | I | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 26 | -1 | 11 | 9 | 2 | 5 | 8 | 2 |  | 1 | 6 | -2 | 7 | 2 |  | -1 | 5 | 26 |  | 2 | 26 |
| N | O | T |  | S | $\bigcirc$ | H | A | R | D |  | A | F |  | T | E | R |  | A | L | L |
| 5 | -1 | 7 |  | 26 | -1 | 38 | -2 | 3 | 16 |  | -2 | 4 |  | 7 | 20 | 3 |  | -2 | 11 | 11 |

Solve each of these equations and write the solution in the space provided. Then use your grid as a codebreaker to find the missing message above. The first equation has been done to start you off.

| Letter | Equation | $\boldsymbol{x}$ | Letter | Equation | $\boldsymbol{x}$ |
| :---: | :--- | :---: | :---: | :--- | :---: |
| $\mathbf{A}$ | $6 x+5=-7$ | -2 | $\mathbf{D}$ | $2 x-1=x+15$ | 16 |
| $\mathbf{E}$ | $5 x-4=96$ | 20 | $\mathbf{F}$ | $7 x-3=4 x+9$ | 4 |
| $\mathbf{G}$ | $x+8=3 x-8$ | 8 | $\mathbf{H}$ | $x-19=19$ | 38 |
| $\mathbf{I}$ | $3-4 x=x-7$ | 2 | $\mathbf{L}$ | $x-8=3$ | 1 I |
| $\mathbf{N}$ | $2 x-6=4$ | 5 | $\mathbf{O}$ | $1-8 x=2 x+11$ | - I |
| $\mathbf{Q}$ | $9 x-6=5 x-2$ | I | $\mathbf{R}$ | $6 x-9=2 x+3$ | 3 |
| $\mathbf{S}$ | $5 x+65=8 x-13$ | 26 | $\mathbf{T}$ | $3 x-4=2 x+3$ | 7 |
| $\mathbf{U}$ | $7 x-5=6 x+1$ | 6 | $\mathbf{V}$ | $5 x-5=2 x+22$ | 9 |

Now try making a similar puzzle for your friends to solve.

Y9) 26 LINEAR EQUATIONS

## Equations crossnumber

Complete the puzzle by solving the equations which make up the clues.

|  | ${ }^{1} 2$ | ${ }^{2} 4$ |  | ${ }^{3}$ । | 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{4} 6$ |  | 3 | ${ }^{6} 2$ | 3 |  | 72 |
| ${ }^{8} 4$ | $2$ |  | 0 |  | ${ }^{10}$ । | 0 |
|  | ${ }^{11}$ । | 5 |  | ${ }^{12} 2$ | 2 |  |
| $13$ | I |  | ${ }^{14}$ । |  | ${ }^{15} 4$ | ${ }^{16} 6$ |
| 5 |  | ${ }^{17} 3$ | 9 | ${ }^{18} 3$ |  | 0 |
|  | ${ }^{19} 5$ | 0 |  | ${ }^{20} 4$ | 4 |  |

Clues Across
I $\frac{x}{3}=8$
$35 x+6=56$
$53 x=969$
$8 \frac{x}{7}=6$
$103 x+20=7 x-20$
II $2 x-14=x+1$
I2 $10 x-20=5 x+90$
13 $x+9=40$
I5 $6 x-6=2 x+178$
$17 \frac{x}{3}=131$
$194 x=200$
$202 x-4=84$

Clues Down
$2 x+43=2 x$
$32 x+5=3 x-8$
$4 \frac{x}{4}=16$
$64 x+30=9 x-70$
$78 x+10=7 x+30$
$92 x=422$
$105 x=3 x+248$
13 $\frac{x}{5}=7$
I4 $3 x+3=60$
I6 $10 x=600$
$173 x+3=93$
I8 $2 x+2=70$

## Mathematical words

Complete the puzzle by solving the equations which make up the clues.


When you have finished you should have a collection of mathematical words running horizontally across the grid. The highlighted squares give another word running vertically downwards.

| $\mathbf{A}$ | $2 x+3=37$ | 17 | $\mathbf{H}$ | $\frac{x}{2}=6$ | 12 | $\mathbf{Q}$ | $2 x+3=25$ | 11 |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :--- | :--- |
| $\mathbf{B}$ | $x-9=11$ | 20 | $\mathbf{I}$ | $9 x-6=12$ | 2 | $\mathbf{R}$ | $\frac{x}{2}+7=15$ | 16 |
| $\mathbf{C}$ | $x+26=36$ | 10 | $\mathbf{L}$ | $2 x+15=45$ | 15 | $\mathbf{S}$ | $3 x-14=28$ | 14 |
| $\mathbf{D}$ | $3 x+12=21$ | 3 | $\mathbf{M}$ | $x-12=-11$ | 1 | $\mathbf{T}$ | $x+9=18$ | 9 |
| $\mathbf{E}$ | $2 x-6=8$ | 7 | $\mathbf{N}$ | $3 x+2=20$ | 6 | $\mathbf{U}$ | $x-9=9$ | 18 |
| $\mathbf{F}$ | $x+14=33$ | 19 | $\mathbf{O}$ | $2 x-3=23$ | 13 | $\mathbf{V}$ | $\frac{x}{2}-2=2$ | 8 |
| $\mathbf{G}$ | $4 x-13=3$ | 4 | $\mathbf{P}$ | $3 x+5=20$ | 5 |  |  |  |

## Number patterns

Here is a reminder of some common number patterns.

| 1 | 3 | 5 | 7 | 9 | $\ldots$ | Odd numbers |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 4 | 6 | 8 | 10 | $\ldots$ | Even numbers |
| 1 | 4 | 9 | 16 | 25 | $\ldots$ | Square numbers |
| 1 | 8 | 27 | 64 | 125 | $\ldots$ | Cube numbers |
| 2 | 3 | 5 | 7 | 11 | $\ldots$ | Prime numbers |
| 1 | 2 | 4 | 8 | 16 | $\ldots$ | Powers of 2 |
| 1 | 3 | 6 | 10 | 15 | $\ldots$ | Triangular numbers |

I Write down the next four odd numbers. Are any of these also square numbers?

2 Write down the next four even numbers. Are any of these also square numbers?

I2, 14, I6, I8; YES - I6
3 Find the next four square numbers.
36, 49, 64, 81

4 Find a number (bigger than 1) which is a square number and also a cube number.

5 Find the next five prime numbers.
13, 17, 19, 23, 29
6 How many prime numbers are also even?
---..........-ONE $\qquad$
7 Find the next five powers of two.
32, 64, I28, 256, 512

8 How many powers of two are also prime? $\qquad$ ONE $\qquad$
9 Find the next five triangular numbers.
21, 28, 36, 45, 55
10 Do most of the triangular numbers seem odd, or are most of them even? Equal nos of each
II Is it possible to have a square number which is also a power of 2?
YES e.g 16
$\mathbf{1 2}$ Is it possible to have a prime number which is also a square number? $\qquad$
$\qquad$

## Position-to-term and term-to-term rules

Find the first four terms of the number patterns given by these position-to-term rules.
I $u_{n}=3 n+4$
$7,10,13,16$
$2 u_{n}=5 n-2$
$3,8,13,18$
$3 u_{n}=n^{2}+1$
2,5,10,17
$4 u_{n}=3 n(n+1)$
6, $18,36,60$
$5 u_{n}=n$
1, 2, 3,4
$6 u_{n}=n+n^{2}$
$2,6, .12,20 \ldots \ldots$
$7 u_{n}=10-2 n$
8, 6, 4, 2
$8 u_{n}=2^{n}$
2, 4, 8, 16........
$9 u_{n}=n^{2}+2 n+1$
4,9, 16, 25
$10 u_{n}=n^{2}-2 n+1$
$0,1,4,9$

Find the first four terms of the number patterns given by these term-to-term rules.
I| $u_{n+1}=3 u_{n}-1$ and $u_{1}=1 \quad 1,2,5,14 \ldots \ldots$
l2 $u_{n+1}=5-2 u_{n}$ and $u_{1}=2$
$2,1,3,-1$

I3 $u_{n+1}=u_{n}\left(u_{n}+2\right)$ and $u_{1}=11,3,15,255$
$14 u_{n+1}=100-u_{n}$ and $u_{1}=10 \quad 10,90,10,90$

I5 $u_{n+1}=4 u_{n}+5$ and $u_{1}=1$
$1,9,41,169$
I6 $u_{n+1}=u_{n}-1$ and $u_{1}=10$
$10,9,8,7$

I7 $u_{n+1}=2 u_{n}$ and $u_{1}=1$
$1,2,4,8$
$18 u_{n+1}=100 \div u_{n}$ and $u_{1}=2$
$2,50,2,50$

19 $u_{n+1}=u_{n}^{2}$ and $u_{1}=3$
3, $9,81,6561$
$20 u_{n+1}=u_{n}^{2}$ and $u_{1}=1$
$1,1,1,1$

Find the first three terms of each of these number patterns. State also whether you have been given a position-to-term rule or a term-to-term rule in each case.

21
$u_{n+1}=200 \div u_{n}$ and $u_{1}=2$
2, 100, 2 TTT
$22 u_{n}=50+5 n$
55, 60, 65 PTT
$23 u_{n+1}=100 \div 2 u_{n}$ and $u_{1}=22,25,2 \ldots$ TT
$24 u_{n+1}=100 \div u_{n}$ and $u_{1}=20$
20, 5, 20 TTT
$25 u_{n}=n(n+1)$
2, 6, I2 PTT

## Finding a rule for the $n$-th term

Find a position-to-term rule for each of these linear number patterns.
I $6,8,10,12,14, \ldots$
$2 n+4$
3 20, 18, 16, 14, 12, $-2 n+22$
$25,8,11,14,17, \ldots$
$3 n+2$
$41,8,15,22,29, \ldots$
$7 n-6$
$510,9,8,7,6, \ldots$ $\qquad$ $6101,103,105,107,109$, $2 n+99$
7 11, 22, 33, 44, 55, IIn $\qquad$ $816,13,10,7,4, \ldots$
-....-.-3n.+.. 19

9 $-4,-1,2,5,8, \ldots$
$3 n-7$
$1090,81,72,63,54, \ldots$
$-9 n+99$

Some of these number patterns are linear.
For those that are linear, find a position-to-term rule.
For those that are not, describe the pattern in words.
II $13,16,19,22,25, \ldots \ldots \ldots+\ldots$
I2 $13,14,16,19,23, \ldots$
....up.1. 2, 3, etc.
I3 $1,2,4,8,16, \ldots$
doubling $\qquad$ I4 $1,3,5,7,9, \ldots$
$2 n-1$

I5 $1,3,6,10,15, \ldots$
triangular nos
I6 40, 38, 36, 34, 32, ...
$-2 n+42$
I7 40, 20, 10, 5, 2.5, .. .............alving
I8 $1,4,7,10,13, \ldots$
$3 n-2$
$191,4,9,16,25, \ldots$
square nos.
20 100, $95,90,85,80, \ldots$
$-5 n+105$

21 I start a savings account with $£ 20$, and then add $£ 10$ every month. How
much will I have after $m$ months? Use your formula to work out how much I have after 12 months.
$£ 20+10 \mathrm{~m}$
fl40

22 An old recipe book says I should cook a turkey for 20 minutes, plus an extra 20 minutes for each pound in weight. Find a formula for the cooking time for a turkey weighing $p$ pounds. $20+20 p$

23 A container holds 500 grams of sugar, and I use 30 grams every day. How much sugar will I have left after $d$ days? How many days will it be until I need to refill the container?

500-30d
16 or 17 days

## Number patterns word search



| F | Y | S | A | M | O | V | A | R | T | I |  |  | E |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | R | T | I | G | R | E | T | T | O | R |  |  | L |  |
| D | E | R | V | O | K | F | C |  |  | A | S |  | U |  |
| O | R | B | O | B | W | R | S |  | O |  |  |  |  |  |
|  | S | O | U |  |  |  | I |  | Q |  |  |  | K |  |
|  | E | Y | V |  | A |  | D |  |  |  |  |  | E |  |
|  | F | A | L | L | Y |  |  |  |  |  |  |  |  |  |
|  | H | A | K | E | S | U | R |  | V |  | B |  | T | H |
|  | N | R | 1 | T | R | O | O |  | O | N | A | H | G |  |
|  | S | D |  |  |  |  | P |  |  |  | X |  | E |  |
|  | L | V | I | D | D | A | H |  | K |  | E | T | R | G |
|  | E | L | E | U | G | S | D | K | U | R | D | S | Y |  |
|  | L | A | R | B | I | P | O | S |  |  |  |  | N |  |
|  | A | O | T | A | D | U | Y | L | K | E | S |  | O |  |
|  | H | F | L | S | P | O | B | E | L | H |  |  | R |  |

The wordsearch grid contains twelve words that have something to do with number patterns. Find the words, which may appear forwards, backwards or diagonally. Make sure that you understand the meaning of each word.
The target list of words, in dictionary order, is given below.


## Fibonacci numbers

This sequence of numbers is known as the Fibonacci sequence:

$$
I, I, 2,3,5,8,13, \ldots
$$

Each term is calculated by adding together the two previous ones.
I Make a list of the first fifteen Fibonacci numbers.I, I, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610
2 Look at the third, sixth and ninth numbers. What do you notice. Can you explain EVEN why this happens? ODD + ODD = EVEN

3 Find the sum of the first three terms, and compare your answer with the fifth term.

4-5
4 Find the sum of the first four terms, and compare your answer with the sixth term.

5 Compare your answers to questions 3 and 4. Does there seem to be a general principle?
.........-. 7.8 $\qquad$

Second figure is one larger than the first $\qquad$

6 Using this principle, predict the sum of the first 12 terms of the Fibonacci sequence. Then check this prediction by adding them.

376
7 Take any four consecutive numbers of the Fibonacci sequence. Multiply the two Difference is always outside numbers, and multiply the two inside numbers. What do you notice? Does it matter which block of four consecutive Fibonacci numbers you take? inside, one bigger.
8 The Fibonacci sequence appears frequently in nature, especially when modelling the growth of natural objects or systems. In the diagram below it is being used to make a drawing of a tree.

Complete the drawing


## Rearranging and sketching linear functions

Rearrange these equations into the form $y=m x+c$.
I $2 x+y=3$
$y=-2 x+3$
$2 y+5=4 x$
$y=4 x-5$
$33+6 x=2 y$
$y=3 x+1 \frac{1}{2}$
$4 y-3 x=10$
$y=3 x+10$
$52 x=y-5$
$y=2 x+5$
$6 x+y+1=0 \quad y=-x-1 \ldots$
$710+4 x=2 y$
$y=2 x+5$
$82 x-y-5=0 \quad y=2 x-5$
$93 x+y=1$
$y=-3 x+1$
$102 y-4 x+6=0 \quad y=2 x-3$

Rearrange these equations into the form $y=m x+c$. Then write down the gradient and intercept of each one.
II $y-5=3 x$
$y=3 x+5$
I2 $2 y+3=11 x$
$y=\frac{11}{2} x-\frac{3}{2}$
13 $x-y-4=0 \quad y=x-4$
$14 x+4 y=1$ $y=-\frac{1}{4} x+\frac{1}{4}$
$15 \quad 12 x+4 y=1 \quad y=-3 x+\frac{1}{4}$
I6 $10=x+y$
$y=-x+10$
$1740-10 x=2 y$
$y=-5 x+20$
18
$16-y-5 x=0 y=-5 x+16$
$19 y+3 x=0$
$y=-3 x$
$20 x+y=-10$
$y=-x-10$

The sketch graphs below show four straight line graphs. Find the equation of each one, giving your answer in the form $y=m x+c$.

21


23


22


24


## Keep smiling...

Complete these tables of three quadratic functions.
I $y=x^{2}-10 x+25$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x^{2}$ | 0 | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |
| $-10 x$ | 0 | -10 | -20 | -30 | -40 | -50 | -60 | -70 | -80 | -90 | -100 |
| +25 | +25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |
| $y$ | 25 | 16 | 9 | 4 | 1 | 0 | 1 | 4 | 9 | 16 | 25 |

2
$y=-0.2 x^{2}+2 x+25$

| $x$ | 0 | 2 | 4 | 6 | 8 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $-0.2 x^{2}$ | 0 | $-0 \cdot 8$ | $-3 \cdot 2$ | $-7 \cdot 2$ | $-12 \cdot 8$ | -20 |
| $+2 x$ | 0 | 4 | 8 | 12 | 16 | 20 |
| +25 | 25 | 25 | 25 | 25 | 25 | 25 |
| $y$ | 25 | $28 \cdot 2$ | $29 \cdot 8$ | $29 \cdot 8$ | $28 \cdot 2$ | 25 |

$3 y=0.5 x^{2}-5 x+18$

| $x$ | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $0.5 x^{2}$ | $4 \cdot 5$ | 8 | $I 2 \cdot 5$ | 18 | $24 \cdot 5$ |
| $-5 x$ | -15 | -20 | -25 | -30 | -35 |
| +18 | 18 | 18 | 18 | 18 | 18 |
| $y$ | $7 \cdot 5$ | 6 | $5 \cdot 5$ | 6 | $7 \cdot 5$ |

Now plot the three curves on the grid below.


Finally, add three small circles of radius I unit, centred at $(3,20),(5,13)$ and $(7,20)$.
You might want to use colouring pencils to finish the picture off.

## Every picture tells a story....

The six graphs below each illustrate how something changes over time.
Match the six graphs with these six scenarios.
I The depth of water at a pier as the tide changes over a 24 -hour period. $\qquad$
B
2 The number of people in a football stadium during a four-hour period. $\qquad$ D $\qquad$
3 The amount of fuel in a car's tank during a very long motorway journey. $\qquad$
E $\qquad$
4 The population of the world over a period of 10 years. $\qquad$
A
5 The temperature of a cup of hot coffee as it left to cool down for half an hour.
F

6 The value of a company's shares on the stock market over a period of two weeks. $\qquad$ C
A

B

C

D

E

F


Now make up some similar stories and graphs, and challenge your classmates to match them up.

Y9) 29 QUADRILATERALS, POLYGONS AND ANGLES

## Quadrilaterals and other polygons

路
The box below reminds you of the names of some special quadrilaterals.
(I) ARROWHEAD
(4) KITE
(6) PARALLELOGRAM
(2) RECTANGLE
(5) RHOMBUS
(7) SQUARE
(3) TRAPEZIUM

I From the quadrilaterals in this list, write down all those that have:
a) four equal sides 5 and 7
b) two pairs of equal angles (2), 5, 6, (7)
c) four equal angles 2 and 7
d) opposite sides parallel $2,5,6,7$
e) two pairs of equal sides $I, 2,4,(5), 6,(7)$
f) exactly one pair of parallel sides. 3

The next part of this exercise is about quadrilaterals and other polygons too.
2 Try to draw quadrilaterals with:
a) no line of symmetry
d) three lines of symmetry
b) one line of symmetry
e) four lines of symmetry
c) two lines of symmetry
f) five lines of symmetry.
What do you notice?
c) Two lines of symmetry
a) No line of symmetry

d) Three lines of symmetry not possible
b) One line of symmetry


The possible lines of symmetry are I, 2, 4. These are factors for 4.
e) Four lines of symmetry

f) Five lines of symmetry not possible

Y9) 29 QUADRILATERALS, POLYGONS AND ANGLES

## Quadrilaterals and other polygons (continued)

3 Try to draw triangles with:
a) no line of symmetry
c) two lines of symmetry
b) one line of symmetry
d) three lines of symmetry.

What do you notice?
a) No line of symmetry

b) One line of symmetry

c) Two lines of symmetry not possible
d) Three lines of symmetry

The possible lines of symmetry are $I, 3$. These are the factors for 3.

4 Without making any drawings, try to predict whether it is possible to draw hexagons with:
a) no line of symmetry
e) four lines of symmetry
b) one line of symmetry
f) five lines of symmetry
c) two lines of symmetry
g) six lines of symmetry.
d) three lines of symmetry

Now make some drawings to see if you were right.
a) yes
b) yes
c) yes
d) yes
e) $n o$
f) $n o$
g) yes

Y9) 29 QUADRILATERALS, POLYGONS AND ANGLES

## Quadrilaterals and other polygons (continued)

a) No line of symmetry

d) Three lines of symmetry

b) One line of symmetry

g) Six lines of symmetry

c) Two lines of symmetry


5 Investigate the number of lines of symmetry for other polygons.
(Y9) 29 QUADRILATERALS, POLYGONS AND ANGLES


Missing angles
Find the angles marked with letters.


- $70^{\circ}$

-----------

$$
f=60^{\circ}, g=50^{\circ}, h=70^{\circ}
$$



$$
o=45^{\circ}, p=135^{\circ}-q=45^{\circ}, r=135^{\circ}
$$


$280^{\circ}$

$$
t=115^{\circ}, u=65^{\circ}, v=115^{\circ}
$$


(C) Letts Educational 2002
$\qquad$

## Quadrilaterals wordsearch

The wordsearch grid contains fifteen words that have something to do with quadrilaterals.
The words may run forwards, backwards or diagonally.
Write down each word as you find it, and explain its meaning.
Write down all the special properties of each quadrilateral as you find it.

| C |  | T | A | R | E | S | W |  | P | V | A | N | Q | O |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y |  | A | N | G | L | E | Z |  | R | E | U | O | U | R |
| A |  | K | G | D | A | E | H |  | W | O | R | R | A | T |
|  |  | M | I | L | S | Q | D |  | C | I | S | L | D | M |
|  |  | E | C | T | A | N | G |  | L | E | U | X | R | U |
|  |  | P | S | C | E | D | R |  | M | C | B | E | I | E |
|  |  | R | Y | F | C | U | B |  |  | E | M | L | L | T |
|  |  | Z | M | L | O | V | D |  | X | I | O | Q | A | I |
|  |  | F | M | F | L | N | P |  | J | P | H | N | T | S |
|  |  | U | E | S | E | S | Q |  | U | A | R | E | E | O |
|  |  | B | T | P | D | R | R B |  | P | K | L | C | R | P |
| M |  | A | R | G | O | L | E |  | L | L | A | R | A | P |
| B |  |  | Y | E | O | K | C |  | E | Q | U | A | L | O |
| P |  | A | R | A | L | L | E |  |  | I | N | C | D | J |
|  | Nate | $\underset{\text { WHEAD }}{E}$ |  |  |  |  |  |  |  |  |  |  |  |  |

Translations and reflections

a) Translate the letter $F$ shape by $\binom{5}{-1}$
b) Reflect the letter F in the line $x=7$.

2

a) Reflect the letter L in the line $y=0$.
b) Translate the letter L by $\binom{-5}{-3}$.

3


Look at the three T-shapes, labelled $\mathrm{T}_{1}, \mathrm{~T}_{2}, \mathrm{~T}_{3}$.
a) Describe a transformation that will take $\mathrm{T}_{1}$ to $\mathrm{T}_{2}$. translation $\binom{5}{-1}$
b) Describe a transformation that will take $\mathrm{T}_{1}$ to $\mathrm{T}_{3}$. reflection in. $y=5 \ldots$

4 A triangle $T_{1}$ is translated by $\binom{5}{-1}$ to form an image triangle $T_{2}$. Triangle $T_{2}$ is then given a further translation of $\binom{6}{4}$ to form another image at $T_{3}$.
a) Describe a single transformation which would take $\mathrm{T}_{1}$ to $\mathrm{T}_{3}$.
translation
b) What transformation would take $\mathrm{T}_{3}$ to $\mathrm{T}_{1}$ ? translation


## Rotations


a) Rotate the shape A through $90^{\circ}$ clockwise about the origin. Label the result B.
b) Rotate the shape A through $90^{\circ}$ anticlockwise about the origin. Label the result C.
c) What single transformation would take shape B directly to shape C ? $180^{\circ}$ rotation about origin

3


2

a) Rotate the shape $P$ through $90^{\circ}$ clockwise about the point $(1,0)$. Label the result Q .
b) Rotate the shape $P$ through $90^{\circ}$ anticlockwise about the point $(2,-3)$. Label the result R.
c) What single transformation would take shape Q directly to shape R ? $180^{\circ}$ rotation about $(0,-2)$

The diagram shows five triangles labelled
$\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E .
a) Describe fully the rotation that will take triangle A onto triangle B. ${ }_{(3,5)}^{80^{\circ}}$ rotation about
b) Describe fully the rotation that will take triangle A onto triangle D. rotation about the origin
c) Are the five triangles all congruent to each other? Yes, because they are the same shape and size. (They are identical.)

## Not a clue - a crossword with symmetry

Before solving this crossword you must complete the grid. It has both a vertical and a horizontal line of symmetry, so you have to shade in 19 more squares.
Once you have done that, you have to fit the words into the grid. The words round the outside have rotational symmetry order 2.

| S | E | R | I | F |  | P | A | R | T | S |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P |  | O |  | A | C | E |  | U |  | L |
| O | C | T | E | T |  | N | O | B | L | E |
| O |  | O |  | E | A | T |  | I |  | E |
| L | O | R | D |  | N |  | P | A | C | K |
|  | N |  | U | T | T | E | R |  | O |  |
| K | E | R | B |  | R |  | Y | E | L | L |
| E |  | A |  | P | E | T |  | N |  | O |
| E | L | D | E | R |  | I | N | T | R | O |
| L |  | A |  | O | F | F |  | E |  | P |
| S | T | R | A | P |  | F | I | R | E | S |

Despite the name of this puzzle, here is a clue if you are stuck. The first word across coded in Atbash code is HVIRU. In Atbash code, you replace A by $\mathrm{Z}, \mathrm{B}$ by $\mathrm{Y}, \mathrm{C}$ by X , etc. You can decode the clue if you need to.

| The words |  |  |  |
| :--- | :--- | :--- | :--- |
| 3-letter | 4-letter | 5-letter |  |
| ACE | FATE | ANTRE | PARTS |
| COL | KERB | ELDER | RADAR |
| DUB | LORD | ENTER | ROTOR |
| EAT | PACK | FIRES | RUBIA |
| OFF | PENT | INTRO | SERIF |
| ONE | PROP | KEELS | SLEEK |
| PET | TIFF | LOOPS | SPOOL |
| PRY | YELL | NOBLE | STRAP |
|  |  | OCTET | UTTER |

## Rangoli patterns

Rangoli is a traditional Hindu form of decoration, forming patterns of coloured sand and riceflour.
Here is one way to make Rangoli patterns using square dotty paper.

Step 1 Draw a frame. Mark vertical and horizontal lines as shown.


Step 3 Reflect the pattern using the vertical and horizontal lines as mirror lines.

Step 2 Join some of the dots in one of the quarters.


Step 4 Finish by reflecting each pattern so that the diagonals are also mirror lines.


Now try making some Rangoli patterns of your own on the grids below.
Colour them in so that the vertical, horizontal and diagonal lines are mirror lines.


Y9 31 THE CIRCLE

## Circle problems

I A car wheel has a diameter of 0.5 metre．Find its circumference． 1.57 m
2 The diameter of a milk bottle is 7.5 cm ．Find its circumference． 23.6 cm
3 A semicircle has a diameter of 15 m ．
a）Find its area．$\quad 88.4 \mathrm{~m}^{2}$
b）Find the perimeter of the semicircle．
38.6 m

4 A trundle wheel rotates once for every metre it is pushed．Calculate its radius correct to one decimal place． 15.9 cm

5 The wheels of Sarah＇s bike have a diameter of 45 cm ．
a）What is the circumference
of each wheel？$\quad 141.4 \mathrm{~cm}$
b）How many complete turns do the wheels make during a 3 km bike ride？
2122

6 A craftsman is making a circular mirror with radius 25 cm ．
a）What area of glass is needed for the mirror？$\quad 1963 \mathrm{~cm}^{2}$
b）What length of plastic would be needed to frame the mirror？$\quad 157 \mathrm{~cm}$

7 A circular flowerbed has a radius of 3 metres．A gardener wishes to plant eight seedlings per square metre of the flowerbed．
a）Find the area of the flowerbed．
$28 \cdot 3 \mathrm{~m}^{2}$
b）Find the number of seedlings the gardener is able to plant．226

8 An ornamental pond is made in the following shape：
a）Find the area of the pond．$\quad 86.1 \mathrm{~m}^{2}$
b）Find the perimeter of the pond． 39.4 m


9 A can of baked beans has a diameter of 10 cm ．The label on the can overlaps itself by 10 mm ，and has a height of 12 cm ．
a）Find the circumference of the can． 31.4 cm
b）Find the area of the label． $389 \mathrm{~cm}^{2}$

IO A washer is made by stamping out a circular disc of radius 4 cm ．
A circular hole of radius 1.5 cm is then removed from the centre of the disc．
Find the area of metal used in the washer．$\quad 43 \cdot 2 \mathrm{~cm}^{2}$


Missing message

(20 | $T$ | $H$ | $E$ |
| :---: | :---: | :---: |
| 16.9 | 0.2 | 8.1 |

| $P$ | $E$ | $R$ | $I$ | $M$ | $E$ | $T$ | $E$ | $R$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25.1 | 8.1 | 22.6 | 2.5 | 2.8 | 8.1 | 16.9 | 8.1 | 22.6 |


| O | F |
| :---: | :---: |
| 24.6 | 3.9 |
| A |  |
| 37.7 |  |$\quad$| C | I | R | C | L | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 7.1 | 2.5 | 22.6 | 7.1 | 28.3 | 8.1 |


| I | S |
| :---: | :---: |
| 2.5 | 22.9 |


| C | A | L | L | $E$ | $D$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 7.1 | 37.7 | 28.3 | 28.3 | 8.1 | 15.7 |


| I | T | S |
| :---: | :---: | :---: |
| 2.5 | 16.9 | 22.9 |


| C | I | R | C | U | M | F | E | R | E | N | C | E |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7.1 | 2.5 | 22.6 | 7.1 | 5.0 | 2.8 | 3.9 | 8.1 | 22.6 | 8.1 | 19.9 | 7.1 | 8.1 |

Solve each of the problems below. Work to one decimal place, and take $\pi=3.142$. Then substitute the matching letters into the boxes above to discover the missing message.

| A | The circumference of a circle if its radius is 6 cm. | $37 \cdot 7$ |
| :--- | :--- | :--- |
| C | The area of a circle if its radius is 1.5 m. | $7 \cdot 1$ |
| D | The circumference of a circle with diameter 5 cm. | $15 \cdot 7$ |
| E | A circle has a circumference of 25.6 cm. Find its diameter. | 8.1 |
| F | A circle has an area of $49 \mathrm{~m}^{2}$. Find its radius. | 3.9 |
| H | Area of a circle with diameter 0.5 cm. | 0.2 |
| I | Find the circumference of a circle with radius 0.4 mm. | 2.5 |
| L | Find the area of a circle with diameter 6 cm. | $28 \cdot 3$ |
| M | A circle has an area of $25 \mathrm{~cm}^{2}$. Find its radius. | $2 \cdot 8$ |
| N | A circle has a circumference of 125 cm. Find its radius. | $19 \cdot 9$ |
| O | The area of a circle with radius 2.8 m. | $24 \cdot 6$ |
| P | The circumference of a circle with diameter 8 cm. | $25 \cdot 1$ |
| R | The circumference of a circle with radius 3.6 cm | $22 \cdot 6$ |
| S | The radius of a circle with circumference 144 m. | $22 \cdot 9$ |
| T | The diameter of a circle with area $224 \mathrm{~cm}^{2}$. | $16 \cdot 9$ |
| U | The radius of a circle with area $80 \mathrm{~cm}^{2}$. | $5 \cdot 0$ |

Y9) 31 THE CIRCLE

## Round and round in circles

Draw five squares, each of side 8 cm . Then construct accurate copies of the diagrams shown below. The circles, semicircles and quarter-circles either touch the side of the square or touch each other.

I


3


5


2


4


Now calculate the size of the shaded area in each case. $\quad 16 \pi=50.3 \mathrm{~cm}^{2}$
What do you notice? All the same area.
Try to make up some more diagrams of your own with the same property.
$\qquad$

Y9 3I THE CIRCLE

## The London Eye

The British Airways London Eye, also known as the Millennium Wheel, stands on the south bank of the River Thames in central London.
Here is some information about it.

| Diameter | 135 metres |
| :--- | :--- |
| Weight of hub and spindle | 330 tonnes |
| Total weight | 1900 tonnes |
| Weight of a single cable | 1.5 tonnes |
| Speed | $0.26 \mathrm{~m} / \mathrm{s}$ |
| Time to revolve | 30 minutes |
| Viewing distance | 25 miles $/ 40$ kilometres |
| Number of passenger capsules | 32 |
| Capsule capacity | 25 passengers |

Use the information in the table to help you answer these questions.
I Calculate the circumference of the Eye, in metres.
424 m

2 Divide the circumference, in metres, by the speed, in metres per second. This will tell you how many seconds it takes the Eye to rotate once.

1631 sec
27.2 mins

3 Divide your answer to question 2 by 60 . What does this represent? How accurately does it agree with the value given in the table?

4 How many passengers can the Eye carry when it is full?
$=$ time to revolve
The table figure is to the nearest 10 minutes.

800

5 Calculate the total weight of all the passengers, assuming an average passenger weighs 60 kg . Give your final answer in tonnes.

48 tonnes

6 Using your answer to question 5, express the total weight of the passengers as a percentage of the (empty) weight of the Eye.
$2 \cdot 5 \%$
7 The Eye is open to passengers for approximately 12 hours each day. How many passengers could it carry in a single day, if it runs full to capacity throughout the day?

## The London Eye (continued)

8 Multiply your answer to question 7 by 365 . What does this figure represent?

7 million
= max yearly total passengers

9 The price for an adult flight is approximately $£ 10$, and the price for a child is approximately half this. Assuming that a full wheel is made up of equal numbers of adults and children, work out how much the passengers on the wheel at any one time have paid in total.

10 Using your answer to question 9, work out an upper limit for the total revenue that might be earned by the Eye in one year, assuming that it is always full to capacity throughout each 12-hour day.

Here are some general knowledge questions about the Eye. You might want to use an Internet search engine to help you find the answers.

II In which country was the main structure built?
I2 British Steel provided the tubular steel. By what name is this company now known?
corus

I3 In which country were the passenger capsules made?
France

14 On a clear day a passenger could see Heathrow Airport and Windsor Castle, 25 miles away: true or false?
true

I5 What is the name of the gardens at the foot of the Eye?
Jubilee Gardens
I6 In what year was the London Eye first opened?

Y9 32 SURVEYS AND SAMPLING

## Surveys and pie charts

I During last week the school library lent out 40 detective stories, 30 thrillers, 50 science fiction books, 20 romantic novels and 10 factual books. Angles are: $96^{\circ}, 72^{\circ}, 120^{\circ}, 48^{\circ}, 24^{\circ}$ Draw a pie chart to show this information.

2120 people were surveyed about how they travel to work each day. The results are shown in the pie chart.
a) How many people travel by train?
b) How many travel by bicycle?

6 30
c) What angle represents cars?
d) How many people travelled by car?


330 pupils were asked to vote for Form Captain. Angles are: $96^{\circ}, 144^{\circ}, 120^{\circ}$ Andy received 8 votes, Ben 12 and Cath 10.
Illustrate this information in a pie chart.
4 This pie chart shows the costs involved in manufacturing a car costing $£ 8500$.
a) What size angle is the cost of labour? $\qquad$
b) What fraction of the total costs are materials?
c) How much are the overheads?
d) What percentage of the total cost is labour?
e) What are the labour costs?
$\frac{1}{3}$
.-..... $£ 708$
...... $58 \%$
. $£ 4958$


## School survey

Here are some questions used by some Year 9 children for a Statistics project.


I The questions are not all very good. Rewrite them so that they are clearer and more precise, and easier to answer.
You may not need to have 10 separate questions once you have rewritten them.

Other than improved wording, eg Q4, questions need answers to select from, eg on Q3.

2 When you have done this, try the questions out on your classmates. (This is called a pilot survey, and is used to check that questions will work properly.)

The survey is to be carried out on all the Year 9 children at the school. There are 80 girls and 60 boys in the year group.
Damian says: "In the whole school there are roughly equal numbers of boys and girls, so I'm going to choose a sample with 35 boys and 35 girls in it."

Eileen says: "In this year group there are more girls than boys, so I'm going to use a sample of 70 children too, but with more girls than boys."

3 How many girls, and how many boys, should Eileen include in her sample?
40 girls, 30 boys

4 Explain carefully whether you think Damian or Eileen has the better sampling method.

Eileen's is more representative
$\qquad$

Y9 32 SURVEYS AND SAMPLING

## Sports day

This is really a summer term activity.
You need to collect some data about each member of your class.
You could ask your PE teacher to help with data collection, or your mathematics teacher might set up a simple long-jump course.

I Design a record sheet for collecting data about members of your class.
It might look something like this:

| Name | Boy or girl | Inside leg (cm) | Long jump (m) |
| :---: | :---: | :---: | :---: |
| Stacey | $G$ | 51 | 2.3 |
| Dilip | B | 65 | 3.9 |

2 Now ask each member of the class to measure their inside leg, in centimetres. Write the results on your data collection sheet.

3 Each member of the class should also complete the long jump. Once again, record the results on your data collection sheet.

4 Draw a scatter diagram of the results for everyone in your class. Does the graph show any evidence of correlation?

5 Now make two separate scatter diagrams: one for boys, and another for girls. Are there any similarities or differences between the two graphs?

6 Now choose your own sports measurements and collect some data. For example, you might want to investigate pupils' heights and their high jump ability. Another good comparison would be to look at ages and time taken to run 100 metres.
Can you find any correlation? Do you need to consider boys and girls separately?
Write a short report about your investigation, and give a brief talk to the rest of your class about your findings.

## Sample space diagrams

I A spinner shows scores of 1,2 or 3 with equal probability.
A second spinner shows 1 or 2 with equal probability.

a) Copy and complete the sample space diagram to show the total score on the two spinners.
b) Find the probability that the total is 2 . $\qquad$
c) Find the probability that the total is 4 .
$\frac{2}{6}=\frac{1}{3}$
2 A coin is tossed and a die is thrown. Draw a sample space diagram to show all 1 the outcomes. Use your diagram to find the probability of obtaining:

| H | HI | H 2 | H 3 | H 4 | H 5 | H 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T | TI | T 2 | T 3 | T 4 | T 5 | T 6 |

a) a head together with an even number
$\frac{3}{12}=\frac{1}{4}$
b) a tail with a score of less than 3 .

$$
\frac{2}{12}=\frac{1}{6}
$$

3 A square spinner has an equally likely chance of showing 1, 2, 3 or 4 . It is spun, and at the same time a normal die is thrown. The scores are added together.
a) Draw a sample space diagram to show all 24 equally likely outcomes.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |

b) Find the probability that the total is 9 .

$$
\frac{2}{24} \ldots=\frac{1}{12} \ldots
$$

c) Find the probability that the total is 10 .
$\frac{1}{24}$
d) Find the probability that the total is less than 9 .
$\frac{21}{24}=\frac{7}{8}$
4 In a family there are two girls, Anne and Brigita, and two boys, Carlos and David. Two children are chosen at random to go to the video library. Draw a sample space diagram to show all the possible pairs of children, and use it to find the probability that:
a) Anne and Brigita go together $\quad \frac{2}{12}=\frac{1}{6}$
b) a boy and a girl go
$\frac{8}{12}=\frac{2}{3}$
c) David goes.
$\frac{6}{12}=\frac{1}{2}$
$\qquad$

Y9 33 PROBABILITY

## Tree diagrams

I I have seen the weather forecast for the weekend.
The probability that it will rain on Saturday is 0.3 .
The probability that it will rain on Sunday is 0.4 .
Copy and complete the tree diagram to show the different possibilities for the weekend.


Use your tree diagram to find the probability that:
a) it will not rain on either day $\qquad$ b) it will rain on just one day. $\qquad$

2 A triangular spinner has an equally likely chance of scoring 1, 2 or 3. It is spun twice and the scores are added together. Draw a tree diagram to show all the outcomes and probabilities. Then use your diagram to find the probability of:
a) scoring a total of 6
$\frac{1}{9}$
b) scoring a total of 4 . $\frac{3}{9}=\frac{1}{3}$

3 A football team wins, loses or draws each match with probabilities of $0.5,0.3$ and 0.2 respectively. The team plays two matches. Draw a tree diagram to show all the possible outcomes, and then use it to find the probability that:
a) both matches were lost
0.09
b) there was at least one draw.

4 A bag contains five red sweets and three green sweets. Rana chooses a sweet and then replaces it; she then chooses another. Draw a tree diagram to show this, and use it to find the probability that:
a) both sweets are red
$\frac{25}{64}$
b) exactly one sweet is green. $\quad \frac{30}{64}=\frac{15}{32}$

5 Flower seeds in a packet look identical but there are three different colours. Half of them are yellow, one third are red and the rest are white. Draw a tree diagram to show the possible colours of the first two seeds to bloom. Find the probability that the first two flowers are:
a) both white
$\frac{1}{36}$
b) red and yellow, in any order. $\frac{2}{6}=\frac{1}{3}$

## Probability maze



## How to play the Probability maze

- A IOp coin is dropped into the slot at the top of the maze.
- At each junction the coin is equally likely to fall left or right.
- If the coin lands at A or D you win 20p.
- If the coins lands at B or C then you lose.

I Mark probabilities along the branches of the maze.
2 Calculate the probability that the coin ends up at:
a) A $\qquad$ b) $\mathrm{B} \quad \frac{3}{8} \ldots$
c) C $\qquad$ d) D . $\qquad$

3 If you owned the game, approximately how much money would you expect to make after 200 coins had been dropped? $\qquad$
4 Design your own coin maze, and calculate the probabilities of winning different amounts. Challenge your partner to calculate how much money will be made after playing the maze 200 times.

